***Varianta 021***

## SUBIECTUL I

**a)** $13$.
**b)** $\sqrt{82}$.
**c)** $\frac{1}{2}$.
**d)** $3x-4y-25=0$
**e)** $BC=2\sqrt{2-\sqrt{3}}$
**f)** $a=0,\ b=1$.

**SUBIECTUL II**

**1.**
**a)** $\hat{8}$ .
**b)** 0.
**c)** 1.
**d)** $0$.
**e)** $\frac{4}{5}$.

**2.**
**a)** $3x^2+2$ .
**b)** $-\frac{35}{4}$.
**c)** $2$.
**d)** $f'(x)>0$ , $\forall x\in$ R . Atunci funcția $f$ este strict crescătoare pe **R** .
**e)** $\frac{2}{5}$.

**SUBIECTUL III**

**a)** Catul este $X$ iar restul este 1.
**b)** $g(-1)=0$ .
**c)** $x_1x_2x_3x_4=\frac{1}{1}=1$.
**d)** $y^3+y^2+y+1=0$ sau $(y+1)(y^2+1)=0$ ,atunci $y_1=-1$, $y_2=i$, $y_3=-i$.
**e)** $y_1^{2007}+y_2^{2007}+y_3^{2007}=-1$ .
**f)** Din **a)** ,avem $f(X)=Xg(X)+1$. Deci $X\ g(X)=f(X)-1$.
$f(x_1)=f(x_2)=f(x_3)=f(x_4)=0$.

Atunci $g(x_1)g(x_2)g(x_3)g(x_4)=\frac{-1}{x_1}\cdot\frac{-1}{x_2}\cdot\frac{-1}{x_3}\cdot\frac{-1}{x_4}=\frac{1}{x_1x_2x_3x_4}=\frac{1}{1}=1\in$ **N** .

**g)** Din **f) ,** avem $f(X)=Xg(X)+1$.Cum $g(y_1)=g(y_2)=g(y_3)=0$

$f(y_1)+f(y_2)+f(y_3)=3$

**SUBIECTUL IV**

**a)** $f'(x)=e^x\left[ax^2+(2a+b)x+b+c\right]$ .

$f''(x)=e^x\left[ax^2+(4a+b)x+2a+2b+c\right]$ .

**b)** $g_n(0)=\mathrm{e}^0\left(0+n^2\right)=n^2$.

**c)** $g_0'(x)=e^x\left(-x^2-x+1\right)$**.**

**d)**Cum $f(0)=c\Rightarrow c=0$ .Cum $f'(0)=b+c$, iar $f'(0)=1\Rightarrow b=1$ .

Din $f''(0)=4$ obținem $2a+2b+c=4$ .Deci $a=1$ .

**e)**Verificăm propoziția pentru $n=1$.

$P(1): 1^2=\dfrac{1(1+1)(2\cdot 1+1)}{6}$; adevărată .Demonstrăm $P(k)\Rightarrow P(k+1)$.

$P(k): 1^2+2^2+...+k^2=\dfrac{k(k+1)(2k+1)}{6}, \forall k\in \mathbf{N}^*$ **;** adevărată.

$P(k+1): 1^2+2^2+...+k^2+(k+1)^2=\dfrac{(k+1)(k+2)(2k+3)}{6}, \forall k\in \mathbf{N}^*$ **.**

Obținem $\dfrac{k(k+1)(2k+1)}{6}+(k+1)^2=\dfrac{k(k+1)(2k+1)+6(k+1)^2}{6}=$

$=\dfrac{(k+1)(k+2)(2k+3)}{6}, \forall k\in \mathbf{N}^*$ **.**

Deci **,** $1^2+2^2+...+n^2=\dfrac{n(n+1)(2n+1)}{6}$ , $\forall n\in \mathbf{N}^*$ **.**

**f)** $\lim\limits_{n\to\infty}\dfrac{g_1(0)+g_2(0)+...+g_n(0)}{n^3}=\lim\limits_{n\to\infty}\dfrac{n(n+1)(2n+1)}{6n^3}=\dfrac{1}{3}$ .

**h)** $\int_0^1 g_0(x)dx=\int_0^1 e^x\left(-x^2+x\right)dx=\int_0^1\left(e^x\right)'\left(-x^2+x\right)dx=e^x\left(-x^2+x\right)\Big|_0^1-\int_0^1 e^x(-2x+1)dx=$

$=-\int_0^1\left(e^x\right)'(-2x+1)dx=-e^x(-2x+1)\Big|_0^1+\int_0^1 e^x(-2)dx=$

$=\mathrm{e}+1-2e^x\Big|_0^1=-e+3.$